MAT1856/APM466 Assignment 1

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Fundamental Questions - 25 points

1.
   (a) Governments issue bonds to avoid the inflationary costs or printing money, while still creating funds for expenditures that they do not need to pay off until a later maturity date.

   (b) Hypothetically, the long-term portion of the yield curve may flatten because investors find it difficult to differentiate the risks or returns between a 25-year and a 30-year bond due to the high uncertainty of economic conditions (and possible default) far into the future, converging the yields for longer maturities.

   (c) Quantitative Easing (QE) is a monetary policy tool where a central bank commits to injecting liquidity into the economy during times of crisis when cutting interest rates alone is insufficient to stimulate growth. In March 2020, the Federal Reserve implemented QE by pledging to purchase \$500 billion in Treasury securities and \$200 billion in mortgage-backed securities (MBS) to stabilize financial markets, support liquidity, and work toward its 2% inflation target.

2. The Bonds chosen are:

Bond Name	Maturity Date	Coupon
CAN 1.25 March 25	March 2025	1.25%
CAN 0.5 Sept 25	September 2025	0.5%
CAN 0.25 March 26	March 2026	0.25%
CAN 1.0 Sept 26	September 2026	1.0%
CAN 1.25 March 27	March 2027	1.25%
CAN 2.75 Sept 27	September 2027	2.75%
CAN 3.5 March 28	March 2028	3.5%
CAN 3.25 Sept 28	September 2027	3.25%
CAN 4.0 March 29	March 2029	4.0%
CAN 3.5 Sept 29	September 2029	3.5%
CAN 2.75 March 30	March 2030	2.75%

   In order to perform the analysis, I found a series of bonds that each maturing 6 months from each other and chose to use this series since almost all of the bonds were a similar distance from the target dates, meaning the approximation to the current date would be consistent through the graphs. Furthermore, I tried to utilize low coupon bonds when they were available but settled with higher coupon bonds towards the later maturity dates.

3. Principal Component Analysis (PCA) is a helpful tool for understanding patterns in a data set. The eigenvalues represent the amount of variance or spread in each of the components, while the eigenvectors associated with these eigenvalues indicate which components show the most variation.

The "directions" with the most variation are considered the most important, as they best represent the data and are therefore labeled as the "principal" components. In essence, the eigenvectors point to the most important directions in the data, and the eigenvalues tell us how much of the variation in the data is captured by each direction.

Empirical Questions - 75 points

4. Before stating any calculations, there has to be a conversion from clean price (price listed) to dirty price (DP). For this calculation we assume the day we are calculating the curves from is the day the prices were extracted. Due to our choice in bonds, the calculation for dirty price is relatively routine like so:

$$DP_i = Price\_Listed_i + \frac{\text{Days Since Last Coupon}}{365} \cdot \text{CPN Payment}$$

This is due to the choice in bonds. By choosing bonds that are 6 months apart, we are able to guarantee that the number of days since the last coupon is 121 + T where T = 6,7,8,9,10,13,14,15,16,17.

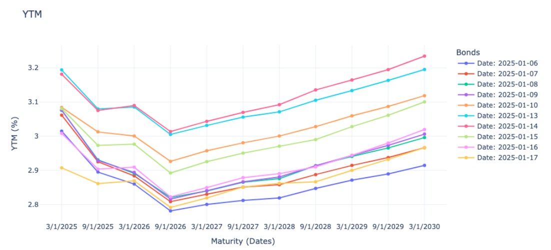
   (a) The formula used to calculate continuously compounded Yield to Maturity (YTM) is:

$$Price = \Sigma_{i=1}^{N} CPN_i \times e^{-YTM \times i} + (CPN_i + FV) \times e^{-YTM \times N}$$

   Since algebraically solving for YTM is tedious especially with 10 or more payments, I used a newton optimizer to calculate the YTM (IRR). By passing in an array with just the price and cash flows, a basic IRR function would assume yearly payments when the actual equation for we are solving for is (I use the CAN 0.25 March 26 Bond as an example):

$$0 = -price + 0.125e^{-YTM \times \frac{1}{6}} + 0.125e^{-YTM \times \frac{4}{6}} + 100.125e^{-YTM \times \frac{7}{6}}$$

   Notice how March 1st is 2 months away from January 1st meaning it is $\frac{1}{6}th$ of a year away from our target date. This value is incremented by 0.5 since all of our bonds have semi annual coupons. The resulting adjusted IRR calculation results in the following curves:



   (b) Calculating the spot rates is very similar to the YTM calculation, with the distinction that the rate is a function of time rather than constant throughout the calculation. Firstly, we turn the maturity from dates to years until maturity as well as the number of cash flow periods remaining. Next, a numpy array for the time periods and cash flows is created. Finally, we check to see if the bond is a ZCB, if so, we calculate it using the following formula:

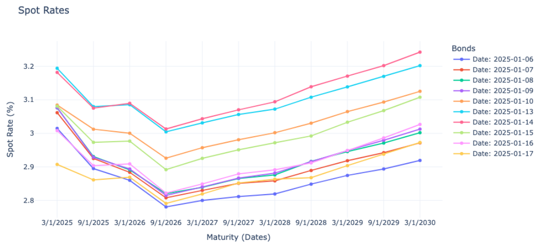
$$\text{ZCB Spot Rate} = \frac{-ln(\frac{price}{100+0.5CPN})}{maturity}$$

   Otherwise, we calculate the present value of the last payment using:

$$\text{PV\_Last\_Payment} = price - \Sigma_{i=1}^{N} CPN \times e^{-r(t_i) \times t_i}$$

   Then we use a similar formula to the ZCB to get the spot rate:

$$\text{Spot Rate} = -\frac{ln(PV\_Last\_Payment)}{(FV+0.5CPN)} \times maturity^{-1}$$

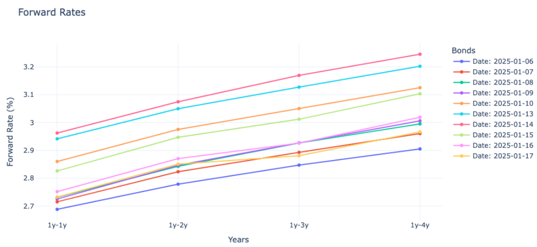


(c) Firstly, since we are looking to find the 1y-Tyr rates where T = 1,2,3,4, we firstly need to find the spot rates at date t in the year 2026, 2027, 2028, 2029, and 2030 where t = 6,7,8,9,10,13,14,15,16,17. By our choice of bonds we computed the spot rates at maturity of the chosen bonds, and now must extrapolate in order to find the spot rates at these dates. We do this by taking a weighted average as follows:

$$\text{New Spot Rate} = \frac{\text{number of days (t - Sept. 1st year T)}}{181} \times \text{September Bond Maturing in X} + \frac{\text{number of days (Mar. 1st year T - t)}}{181} \times \text{March Bond Maturing in X+1}$$

This is just a weighted average based on the distances of the rates calculated to the current day. We made the simplifying assumption of 181 days between Sept. 1st and Mar. 1st each year, ignoring leap years. We then calculate the forward rate with the following formula:

$$\text{Forward Rate (1yr-Tyr)} = \frac{SR_T \times T - SR_1 \times 1}{T-1}$$ where T = 1,2,3,4, and SR_T is the Spot Rate T years after year 1.



5. These are the Covariance Matrix (All values are times 10^{-3}):

	X_1	X_2	X_3	X_4	X_5
X_1	0.470	-	-	-	-
X_2	0.515	0.569	-	-	-
X_3	0.533	0.590	0.614	-	-
X_4	0.556	0.613	0.639	0.674	-
X_5	0.000547	0.601	0.628	0.666	0.661

	F_1	F_2	F_3	F_4
F_1	0.783	-	-	-
F_2	0.724	0.676	-	-
F_3	0.786	0.736	0.821	-
F_4	0.730	0.688	0.770	0.726

6. The first eigenvalue of both matrices is approximately 99% of the trace, indicating its significant role in determining the movement of the yield curve. This eigenvector is linked to shifts in the yield curve, accounting for about 99% of the movement.(All Eigenvalues are multiplied by 10^3)

YTM	1	2	3	4	5
λ	2.965	1.837e-2	3.877e-3	7.063e-5	4.656e-4
% Trace	99.2	0.61	0.13	2.36e-3	1.558e-2
	0.396	-0.396	-0.761	0.327	0.012
	0.436	-0.481	0.157	-0.743	-0.038
	0.454	-0.232	0.586	0.522	0.35
	0.476	0.329	0.121	0.133	-0.796
	0.469	0.671	-0.193	-0.225	0.491

Forward	1	2	3	4
λ	2.97	3.04e-2	7.42e-4	2.96e-3
% Trace	98.8	1.01	2.47e-2	9.86e-2
	0.509	0.662	0.397	-0.381
	0.475	0.294	- 0.402	0.725
	0.524	-0.357	-0.575	-0.518
	0.491	-0.590	0.592	0.246

References and GitHub Link to Code

1. Brookings Institution. Fed Response to COVID-19: Lessons Learned from Monetary and Fiscal Policy. Brookings, 27 Apr. 2022, www.brookings.edu/articles/fed-response-to-covid19/. Accessed 8 Jan. 2025.
2. https://github.com/arielkhait/APM466-A1